Final **Name:** ____________________

Sample

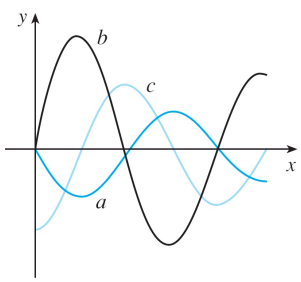
Show your work! You are allowed to use a 4×6 card with notes written on one side and a dedicated calculator.

1. Consider the function $f(x) = x^2 - x$ on the interval $0 \leq x \leq 2$.

   (a) Set up the Riemann sum to approximate the area under the curve. Use $n = 4$ subintervals and select x_i^* to be the right end point of each subinterval.

   (b) Use the Fundamental Theorem of Calculus to get the exact value for the integral.

2. The following figure shows the graphs of f, f', and $\int_0^x f(t)dt$. Identify each graph.



3. Evaluate the following integrals.

   (a) $\displaystyle\int_1^2 (8x^3 + 3x^2)\ dx$

   (b) $\displaystyle\int_1^9 \frac{\sqrt{u} - 2u^2}{u}\ du$

   (c) $\displaystyle\int_0^1 v^2 cos(v^3)\ dv$

   (d) $\displaystyle\int \sin \pi t \cos \pi t\ dt$

   (e) $\displaystyle\int_0^4 |3 - x|\ dx$

   (f) $\displaystyle\int_0^4 \frac{1}{16 + t^2}\ dt$

   (g) $\displaystyle\int \frac{x}{\sqrt{1 - x^4}}\ dx$

   (h) $\displaystyle\int \frac{x + 1}{x^2 + 2x}\ dx$

4. Find the derivative of the following function.

$$F(x) = \int_1^x \sqrt{1+t^4}\, dt$$

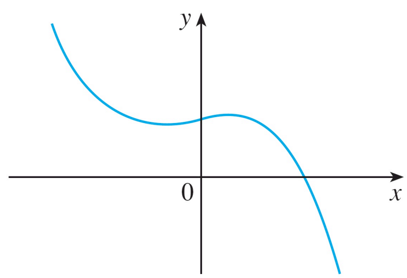
5. A particle moves along a line with velocity function $v(t) = t^2 - t$, where v is measured in meters per second.

   (a) Find the displacement of the particle during the time interval $[0, 5]$.

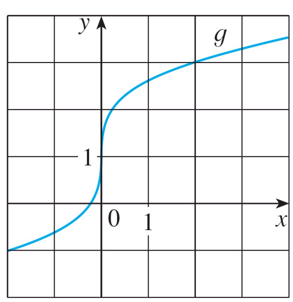
   (b) Find the total distance travelled by the particle during the time interval $[0, 5]$.

6. Find the average value of the function $f(x) = x\sin(x^2)$ on the interval $[0, 2]$.

7. The graph of f is shown. Is f one-to-one? Explain your reasoning.



8. The graph of g is given.



   (a) Estimate the value of $g^{-1}(2)$.

   (b) Sketch the graph if g^{-1}

9. Find the inverse function $f(x) = \dfrac{x+1}{2x+1}$.

10. Find the exact value of each expression.

   (a) $e^{2\ln 3}$

   (b) $\ln e^{\pi}$

   (c) $\log_{10} 25 + \log_{10} 4$

11. Solve the following equations for x.

 (a) $\ln(x+1) + \ln(x-1) = 1$

 (b) $\log_5(c^x) = d$

12. Differentiate the following functions

 (a) $y = \ln(x \ln x)$

 (b) $y = \sqrt{\tan^{-1} x}$

 (c) $f(x) = 4^{2x}$

 (d) $g(x) = \sin^{-1}(4x)$

13. A bacteria culture contains 200 cells initially and grows at a rate proportional to its size. After half an hour the population has increased to 360 cells.

 (a) Find the number of bacteria as a function of time t measured in hours.

 (b) Find the number of bacteria after 4 hours.

 (c) Find the rate of growth after 4 hours.

 (d) When will the population reach 10,000?

14. Simplify the expression.

 (a) $\sin(\tan^{-1} x)$

 (b) $\cos(2 \tan^{-1} x)$

15. Find the following limits using l'Hopital's Rule where appropriate.

 (a) $\displaystyle\lim_{x \to 1} \frac{\ln x}{x-1}$

 (b) $\displaystyle\lim_{t \to \pi/2^+} \frac{\sin x}{\sqrt{x - \pi/2}}$

 (c) $\displaystyle\lim_{x \to \infty} x^3 e^{-x^2}$

 (d) $\displaystyle\lim_{x \to 0} x^2 \ln x$

 (e) $\displaystyle\lim_{x \to \infty} (x - \ln x)$

 (f) $\displaystyle\lim_{t \to 0} (1-3x)^{1/2}$

16. Evaluate the following integrals using integration by parts.

   (a) $\int_0^{\pi/4} x\cos(4x)\ dx$

   (b) $\int t^3 \cos t\ dt$

   (c) $\int e^{\theta} \sin\theta\ d\theta$

17. Find the partial fractions decomposition of the following function.

   $\dfrac{x}{x^2+x-2}$

18. Find the form of the partial fractions decomposition of the following function. It is not necessary to find the coefficients.

   $\dfrac{2x^4-2x^2-1}{x^2(x^2+2)^2(x-3)}$

19. Evaluate the following integrals using a trigonometric substitution followed by other integration techniques.

   (a) $\int \dfrac{1}{\sqrt{9+x^2}}\ dx$

   (b) $\int_0^{\frac{1}{2}} \sqrt{1-4x^2}\ dx$

   (c) $\int \dfrac{\sqrt{x^2-9}}{x^3}\ dx$

20. Evaluate the following integrals using any appropriate technique. Be sure to show how you get your results.

   (a) $\int_0^{\pi/2} \dfrac{\sin x}{1+\cos^2 x}\ dx$

   (b) $\int \cos^3\theta\ d\theta$

21. For each of the following improper integrals either find its value or demonstrate that it is divergent.

    (a) $\displaystyle\int_3^\infty \frac{1}{(x-2)^{3/2}}\,dx$

    (b) $\displaystyle\int_1^2 \frac{1}{(x-2)^{3/2}}\,dx$

    (c) $\displaystyle\int_0^1 x\ln x\,dx$

    (d) $\displaystyle\int_{-1}^1 \frac{e^x}{e^x-1}\,dx$

22. The birth rate of a population is $b(t) = 2200e^{0.024t}$ people per year and the death rate is $d(t) = 1460e^{0.018t}$ people per year. Find the area between these curves for $0 \le t \le 10$. What does the area represent?

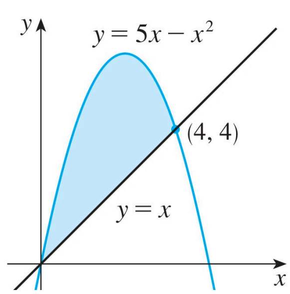
23. Sketch the area enclosed by the givens curves. Find the area of the region.

    (a) $y = (x-2)^2$, $y = x$

    (b) $x = 1 - y^2$, $x = y^2 - 1$

24. Find the volume that results when the area between $f(x) = 1 - x^2$ and the x axis is revolved around the x axis.

25. Consider the area between $y = 5x - x^2$ and $y = x$.

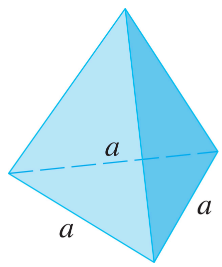


    (a) Set up an integral that could be used to find the volume of the region that results when this area is rotated about the x axis.

    (b) Set up an integral that could be used to find the volume of the region that results when this area is rotated about the y axis.

    (c) Set up an integral that could be used to find the volume of the region that results when this area is rotated about the line $y = -1$.

26. Find an integral that could be used to find the volume of a pyramid with height h and base an equilateral triangle with side a (a tetrahedron). Evaluate the integral.



Trigonometric Identities

(1) $\sin^2\theta + \cos^2\theta = 1$

(2) $1 + \tan^2\theta = \sec^2\theta$

(3) $\sin 2x = 2\sin x\cos x$

(4) $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

(5) $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

(6) $\sin^2 x = \dfrac{1-\cos 2x}{2}$

(7) $\cos^2 x = \dfrac{1+\cos 2x}{2}$

Integrals

(1) $\displaystyle\int \frac{1}{u}\,du = \ln|u| + C$

(2) $\displaystyle\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$

(3) $\displaystyle\int \frac{1}{\sqrt{a^2-u^2}}\,du = \sin^{-1}\frac{u}{a} + C, a > 0$

(4) $\displaystyle\int \frac{1}{u\sqrt{u^2-a}}\,du = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C$

(5) $\displaystyle\int a^u\,du = \frac{a^u}{\ln a} + C, a \neq 1$

(6) $\displaystyle\int \ln u\,du = u\ln u - u + C$

(7) $\displaystyle\int \tan x\,dx = \ln|\sec x| + C$

(8) $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + C$

Derivatives

(1) $\dfrac{d}{dx}(\sin^{-1} x) = \dfrac{1}{\sqrt{1-x^2}}$

(2) $\dfrac{d}{dx}(\cos^{-1} x) = \dfrac{-1}{\sqrt{1-x^2}}$

(3) $\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1+x^2}$

(4) $\dfrac{d}{dx}(\cot^{-1} x) = \dfrac{-1}{1+x^2}$

(5) $\dfrac{d}{dx}(\sec^{-1} x) = \dfrac{1}{|x|\sqrt{x^2-1}}$

(6) $\dfrac{d}{dx}(\csc^{-1} x) = \dfrac{-1}{|x|\sqrt{x^2-1}}$

(7) $\dfrac{d}{dx}(\log_a x) = \dfrac{1}{(\ln a)x}$

(8) $\dfrac{d}{dx}(a^x) = (\ln a)a^x$

Trigonometric Substitutions

(1) $\sqrt{a^2-x^2}$, $x = a\sin\theta$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

(2) $\sqrt{a^2+x^2}$, $x = a\tan\theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

(3) $\sqrt{x^2-a^2}$, $x = a\sec\theta$, $0 \le \theta < \frac{\pi}{2}$ or $\pi \le \theta < \frac{3\pi}{2}$